[REVISION TOOL · JEE MAIN + ADVANCED · CLASS 11]

CONCEPT MAP

SET THEORY

5 TOPICS · 30 CORE IDEAS · ONE SHEET

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A SETS & HOW TO DESCRIBE THEM 01–06

01 WHAT A SET IS
A·01

A set is a well defined collection of distinct objects. “Well defined” means that for any object there is no argument about whether it belongs. Write $x \in A$ if x belongs to A and $x \notin A$ if it does not.

TWO WAYS TO DESCRIBE A SET

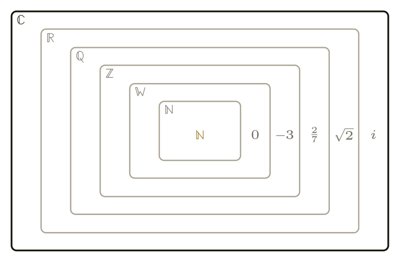
KEY

roster: $A = \{1, 2, 3, 6\}$
builder: $A = \{x : x \mid 6,\ x \in \mathbb{N}\}$

i. Order does not matter: $\{1, 2\} = \{2, 1\}$.
ii. Repeats do not count: $\{1, 1, 2\}$ is the same set as $\{1, 2\}$.
iii. Roster form suits a short explicit list; builder form suits a rule or an infinite set.

“Well defined” is doing real work. The collection of all tall students is not a set, because two people can disagree about a given student. The collection of students above 180 cm is.

02 THE STANDARD NUMBER SETS
A·02



$\mathbb{N} \subset \mathbb{W} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$

SYMBOL	SET
$\mathbb{N}$	natural numbers $1, 2, 3, \ldots$
$\mathbb{W}$	whole numbers, $\mathbb{N}$ together with 0
$\mathbb{Z}$	integers, positive, negative and 0
$\mathbb{Q}$	rationals, p/q with $p, q \in \mathbb{Z}$, $q \neq 0$
$\mathbb{R}$	reals, rationals and irrationals
$\mathbb{C}$	complex numbers $a + ib$

Each one sits inside the next. $\mathbb{N} \subset \mathbb{W} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$, and every inclusion is proper. The irrationals are $\mathbb{R} - \mathbb{Q}$; they are not closed under addition, since $\sqrt{2} + (-\sqrt{2}) = 0$.

03 TYPES OF SET
A·03

i. **Empty set** $\emptyset$ or $\{\ \}$: no elements at all. Note $\{0\}$ and $\{\emptyset\}$ are **not** empty; each has one element.
ii. **Singleton:** exactly one element.
iii. **Finite / infinite:** the counting of elements stops, or it does not.
iv. **Equal sets:** $A = B$ means every element of A is in B and every element of B is in A.
v. **Equivalent sets:** $n(A) = n(B)$. Equal sets are equivalent; equivalent sets need not be equal.
vi. **Comparable:** $A \subseteq B$ or $B \subseteq A$. Otherwise the two are non-comparable.

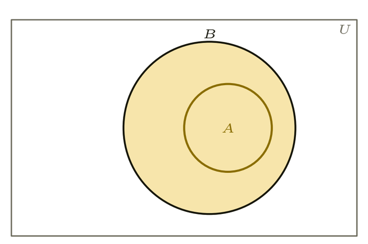
Empty or not? $\{x \in \mathbb{R} : x^2 + 1 = 0\} = \emptyset$, but $\{x \in \mathbb{C} : x^2 + 1 = 0\} = \{i, -i\}$. Always read which universe the builder form is working in.

04 SUBSETS AND PROPER SUBSETS
A·04

If every element of A is also an element of B, then A is a subset of B:

KEY

$$A \subseteq B \iff (x \in A \Rightarrow x \in B)$$



$A \subseteq B$

If in addition $A \neq B$, then A is a **proper** subset, written $A \subset B$.

- Every set is a subset of itself: $A \subseteq A$.
- $\emptyset$ is a subset of every set, including itself.
- $A = B$ exactly when $A \subseteq B$ and $B \subseteq A$ — the standard way to prove two sets equal.

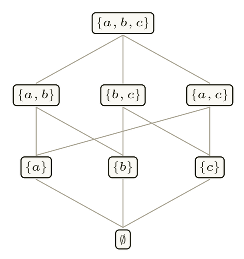
$\in$ and $\subseteq$ are different. For $A = \{1, \{2\}\}$: $1 \in A$ and $\{1\} \subseteq A$, while $\{2\} \in A$ and $\{\{2\}\} \subseteq A$. Mixing the two symbols is the commonest slip in this chapter.

05 POWER SET
A·05

The power set $P(A)$ is the set of **all** subsets of A.

KEY

$$n(A) = m \implies n(P(A)) = 2^m$$

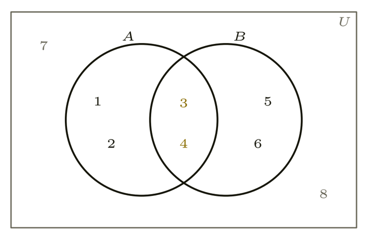


$n(P(A)) = 2^3 = 8$

- $P(A)$ is never empty: it always contains $\emptyset$ and A.
- $P(\emptyset) = \{\emptyset\}$, a set with one element.
- $A \subseteq B \iff P(A) \subseteq P(B)$.
- $X \in P(A)$ means the same thing as $X \subseteq A$.

06 UNIVERSAL SET AND VENN DIAGRAMS
A·06

The universal set U is the set of everything under discussion in the problem. Every other set in that problem is a subset of U.



every element sits in exactly one region

A Venn diagram draws U as a rectangle and each set as a region inside it. Two overlapping sets cut U into 4 regions, three sets into 8.

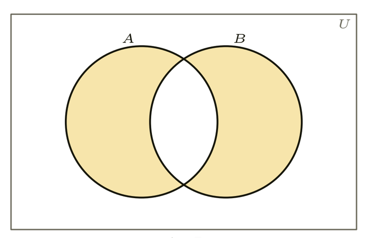
Why the picture is reliable. Every element falls into exactly one region, so counting regions never double counts. That is the whole basis of the counting formulas in Topic D.

B OPERATIONS ON SETS 07–13

07 UNION
B·07

KEY

$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$



$A \cup B$

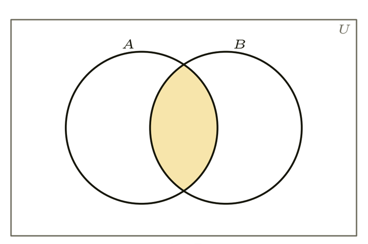
$A = \{1, 2, 3\}$, $B = \{3, 4\}$ give $A \cup B = \{1, 2, 3, 4\}$. The shared 3 is written once.

- $A \subseteq A \cup B$ and $B \subseteq A \cup B$.
- $A \cup \emptyset = A,\ A \cup U = U,\ A \cup A = A$.
- $A \subseteq B \iff A \cup B = B$.

08 INTERSECTION AND DISJOINT SETS
B·08

KEY

$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$



$A \cap B$

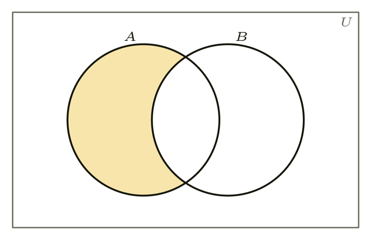
If $A \cap B = \emptyset$ the two sets are **disjoint**: they share nothing.

- $A \cap B \subseteq A$ and $A \cap B \subseteq B$.
- $A \cap \emptyset = \emptyset,\ A \cap U = A,\ A \cap A = A$.
- $A \subseteq B \iff A \cap B = A$.

09 DIFFERENCE
B·09

KEY

$$A - B = \{x : x \in A \text{ and } x \notin B\}$$



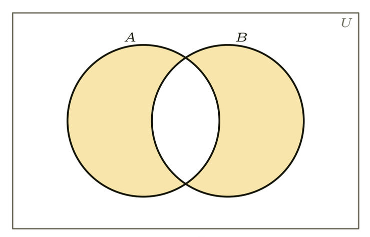
$A - B$

- $A - B$, $A \cap B$ and $B - A$ are three disjoint pieces whose union is $A \cup B$.
- $A - B$ and $B - A$ differ unless $A = B$, so the difference is not commutative.
- $A - B = A \iff A \cap B = \emptyset$, and $A - B = \emptyset \iff A \subseteq B$.
- $(A - B) \cup B = A \cup B$ and $(A - B) \cap B = \emptyset$.

10 SYMMETRIC DIFFERENCE
B·10

KEY

$$A \triangle B = (A - B) \cup (B - A)$$



$A \triangle B$

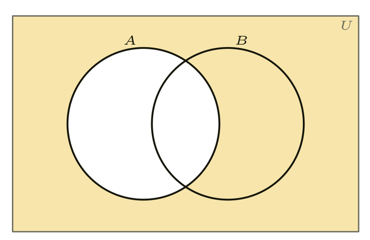
It collects exactly the elements that lie in one of the two sets but not in both, so it is also $(A \cup B) - (A \cap B)$.

- $A \triangle B = B \triangle A$ and $A \triangle A = \emptyset$.
- $A \triangle \emptyset = A$.
- The operation is associative, so $A \triangle B \triangle C$ needs no brackets.

11 COMPLEMENT
B·11

KEY

$$A' = \{x \in U : x \notin A\} = U - A$$



A'

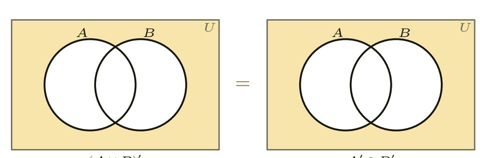
- $A \cup A' = U$ and $A \cap A' = \emptyset$, so A and A' are always disjoint.
- $(A')' = A,\ U' = \emptyset,\ \emptyset' = U$.
- $A \subseteq B \iff B' \subseteq A'$ — taking complements reverses the inclusion.

The complement needs a universal set. Without U the symbol A' has no meaning. Changing U changes A' even though A has not moved.

12 DE MORGAN'S LAWS
B·12

KEY

$$(A \cup B)' = A' \cap B', \qquad (A \cap B)' = A' \cup B'$$



In words: to negate a union, negate each piece and swap $\cup$ for $\cap$. The same two laws hold for any number of sets.

Where marks are lost. $(A \cup B)'$ is not $A' \cup B'$. Test it on $A = \{1\}$, $B = \{2\}$, $U = \{1, 2, 3\}$: the left side is $\{3\}$, the wrong right side is $\{1, 2, 3\}$.

13 DIFFERENCE THROUGH THE COMPLEMENT
B·13

Every difference can be turned into an intersection, which is what makes set expressions simplifiable:

KEY

$$A - B = A \cap B', \qquad B - A = B \cap A'$$

- $A - B = A \iff A \cap B = \emptyset$.
- $n(A - B) + n(A \cap B) = n(A)$.
- $A' - B' = B - A$.
- $(A - B) \cup (A \cap B) \cup (B - A) = A \cup B$, and the three pieces are disjoint.

First move on any hard expression. Rewrite every $-$ as $\cap$ with a complement. What is left uses only $\cup$, $\cap$ and $'$, and then the laws of Topic C apply directly.

C THE ALGEBRA OF SETS 14–18

14 THE BASIC LAWS
C·14

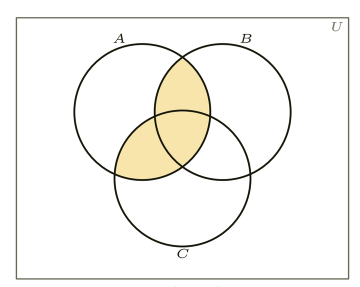
LAW	STATEMENT
Commutative	$A \cup B = B \cup A,\ A \cap B = B \cap A$
Associative	$(A \cup B) \cup C = A \cup (B \cup C)$
	$(A \cap B) \cap C = A \cap (B \cap C)$
Idempotent	$A \cup A = A,\ A \cap A = A$
Identity	$A \cup \emptyset = A,\ A \cap U = A$
Domination	$A \cup U = U,\ A \cap \emptyset = \emptyset$

The difference obeys almost none of these. $A - \emptyset = A$ still holds, but $A - B \neq B - A$ and $(A - B) - C \neq A - (B - C)$ in general. Convert to $\cap$ with a complement first.

15 DISTRIBUTIVE LAWS
C·15

KEY

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$



$A \cap (B \cup C)$

and the one that has no arithmetic analogue,

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Both ways round. With numbers only multiplication distributes over addition. With sets each operation distributes over the other, which is why set algebra has more moves available than ordinary algebra.

16 COMPLEMENT LAWS
C·16

KEY

$$A \cup A' = U, \qquad A \cap A' = \emptyset, \qquad (A')' = A$$

Together with De Morgan these turn any complement of a compound expression into complements of single sets.

$(A \cap (B \cup C))' = A' \cup (B \cup C)' = A' \cup (B' \cap C')$.

Read a prime as “not”. $x \in (A \cap B)'$ says “x is not in both”, which is “x misses A or misses B”, that is $x \in A' \cup B'$. Every set law is a law of logic in disguise.

17 ABSORPTION AND OTHER IDENTITIES
C·17

KEY

$$A \cup (A \cap B) = A, \qquad A \cap (A \cup B) = A$$

- $A \cup (A' \cap B) = A \cup B$ and $A \cap (A' \cup B) = A \cap B$.
- $A \subseteq B \Rightarrow A \cup C \subseteq B \cup C$ and $A \cap C \subseteq B \cap C$.
- $A \triangle B = (A \cup B) \cap (A \cap B)'$.
- $(A \cup B) \cap (A \cup B') = A$.

Absorption is the one to spot. Any time a set appears both alone and inside a bracket joined by the other operation, the bracket collapses and the whole expression is just that set.

18 PROVING TWO SETS ARE EQUAL
C·18

Three routes, in the order to try them:

i. **Algebra.** Rewrite every $-$ as $\cap$ with a complement, then push complements inward with De Morgan and simplify with the laws above.
ii. **Double inclusion.** Take $x \in$ LHS and argue $x \in$ RHS, then reverse. This is the method that always works.
iii. **Venn diagram.** Shade each side. Good for spotting the answer and for a two mark question, but a shaded picture is a check, not a proof.

KEY

$$A = B \iff A \subseteq B \text{ and } B \subseteq A$$

Disproving is faster. To show two expressions are **not** equal, one small counterexample is enough. Try $U = \{1, 2, 3\}$ with $A = \{1, 2\}$, $B = \{2, 3\}$, $C = \{1, 3\}$ before attempting any proof — overlapping test sets, since pairwise disjoint ones let many false identities slip through.

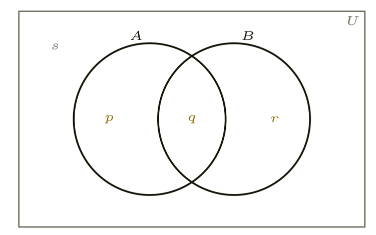
D COUNTING WITH SETS 19–25

19 COUNTING TWO SETS
D·19

$n(A)$ is the number of elements of a finite set A.

KEY

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$



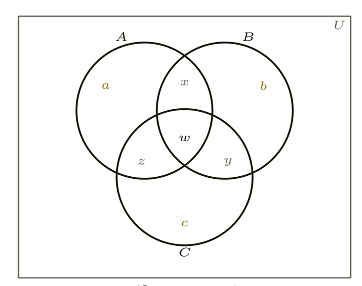
$n(A \cup B) = p + q + r$

- $n(A \cup B) = n(A) + n(B) \iff A$ and B are disjoint.
- $n(A - B) = n(A) - n(A \cap B)$.
- $n(A \triangle B) = n(A) + n(B) - 2\,n(A \cap B)$.
- $n(A' \cap B') = n((A \cup B)') = n(U) - n(A \cup B)$.

20 COUNTING THREE SETS
D·20

KEY

$$\begin{aligned} n(A \cup B \cup C) = {} & n(A) + n(B) + n(C) \\ & - n(A \cap B) - n(B \cap C) \\ & - n(C \cap A) + n(A \cap B \cap C) \end{aligned}$$



exactly one $= a + b + c$

Writing S_1 for the sum of the single counts, S_2 for the sum of the three pairwise counts and S_3 for the triple count, the two formulas are simply

$$n(A \cup B) = S_1 - S_2$$
$$n(A \cup B \cup C) = S_1 - S_2 + S_3$$

Why the signs alternate. Adding $n(A) + n(B) + n(C)$ counts a pairwise overlap twice and the middle three times. Subtracting the pairs removes the middle three times as well, leaving it at zero, so it has to be added back once.

21 EXACTLY ONE, EXACTLY TWO
D·21

With the region names of the figure in section 20:

KEY

$$\begin{aligned} \text{exactly two} &= S_2 - 3S_3 \\ &= n(A \cap B) + n(B \cap C) + n(C \cap A) \\ &\quad - 3\,n(A \cap B \cap C) \\ \text{exactly one} &= S_1 - 2S_2 + 3S_3 \end{aligned}$$

- at least two $= S_2 - 2S_3$
- at least one $= S_1 - S_2 + S_3$
- none of the three $= n(U) - (S_1 - S_2 + S_3)$
- For two sets, exactly one $= n(A) + n(B) - 2\,n(A \cap B) = n(A \triangle B)$.

Correction to the printed sheet. The source gives exactly one as $S_1 - 2S_2 + 3S_3 + 3S_3$, that is with $+6\,n(A \cap B \cap C)$. The coefficient is $+3$. Check it on the picture: adding S_1 counts the middle three times, subtracting $2S_2$ removes it six times, so $+3S_3$ is exactly what brings it back to zero.

22 COUNTING WITH COMPLEMENTS
D·22

When a question asks for “neither”, “none of” or “at most”, count the opposite and subtract:

KEY

$$n(A') = n(U) - n(A)$$

- $n(A' \cap B') = n(U) - n(A \cup B)$ — neither.
- $n(A' \cup B') = n(U) - n(A \cap B)$ — not both.
- $n(A \cap B') = n(A) - n(A \cap B)$ — A only.

Read the English carefully. “Not both” is $(A \cap B)'$; “neither” is $(A \cup B)'$. They are different sets, and a question usually turns on which one is meant.

23 BOUNDS ON THE OVERLAP
D·23

If only $n(A)$, $n(B)$ and $n(U)$ are known, the overlap is not fixed but it is trapped:

KEY

$$\begin{aligned} &\max\{0,\ n(A) + n(B) - n(U)\} \\ &\quad \le\ n(A \cap B)\ \le\ \min\{n(A), n(B)\} \end{aligned}$$

and correspondingly

$$\begin{aligned} &\max\{n(A), n(B)\} \\ &\quad \le\ n(A \cup B)\ \le\ \min\{n(A) + n(B),\ n(U)\} \end{aligned}$$

Both ends are reachable. The overlap is smallest when the two sets are pushed as far apart as U allows, and largest when the smaller set is buried inside the larger. Every value in between occurs, so “find the minimum number of students who take both” is answered by the left bound.

24 THE SURVEY PROBLEM, STEP BY STEP
D·24

i. Name the sets and write down every number the question gives, in the notation $n(\cdot)$.
ii. Draw the Venn diagram and start filling from the **innermost** region $n(A \cap B \cap C)$ outwards.
iii. Each pairwise region gets $n(A \cap B) - n(A \cap B \cap C)$, and each single region is what is left of $n(A)$.
iv. Read the answer off the diagram; use the formulas of sections 20 and 21 only as a check.
v. Verify that all regions plus the outside add up to $n(U)$.

Watch the wording of the data. “25 read A and B” almost always means $n(A \cap B) = 25$, including those who read all three. “25 read A and B but not C” means the pairwise region alone. Deciding this before you start saves the whole question.

25 COUNTING SUBSETS
D·25

For a set with n elements:

KEY

$$\text{subsets} = 2^n, \qquad \text{proper subsets} = 2^n - 1$$

- Subsets with exactly r elements: ${}^nC_r$, and $\sum_{r=0}^{n} {}^nC_r = 2^n$.
- Subsets containing a fixed element: 2^{n-1}; subsets avoiding it: 2^{n-1}.
- Non-empty proper subsets: $2^n - 2$, for $n \ge 1$.
- Pairs (X, Y) with $X \subseteq Y \subseteq A$: 3^n, since each element is in X, in Y only, or in neither.

Where 2^n comes from. Building a subset means answering “in or out?” once per element, independently. That is n decisions with 2 choices each, so 2^n subsets, and the same idea gives 3^n for the nested pair above.

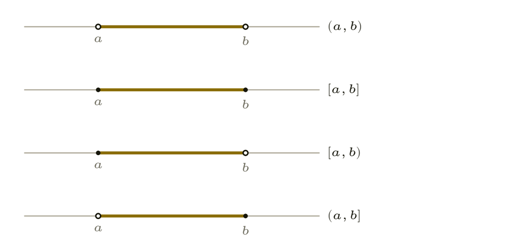
E SETS ON THE NUMBER LINE 26–30

26 INTERVALS
E·26

An interval is a subset of $\mathbb{R}$ with no gaps. A square bracket includes the endpoint, a round bracket excludes it.

KEY

$$(a, b), \quad [a, b], \quad [a, b), \quad (a, b]$$



- For $a < b$ the length of each of the four is $b - a$; only the endpoints differ.
- Unbounded intervals use ∞, which is never included: (a, ∞), $(-\infty, b]$.
- $\mathbb{R} = (-\infty, \infty)$, and a single point $[a, a] = \{a\}$ while $(a, a) = \emptyset$.

27 COMBINING INTERVALS
E·27

Intersect by taking the larger left endpoint and the smaller right endpoint:

KEY

$$[a, b] \cap [c, d] = [\,\max(a, c),\ \min(b, d)]$$

and the result is empty when $\max(a, c) > \min(b, d)$.

- The union of two non-empty intervals is an interval only when they overlap or touch.
- A strict endpoint beats a closed one: $[1, 3) \cap (2, 3] = (2, 3)$.
- $(-\infty, 2] \cup (2, \infty) = \mathbb{R}$, but $(-\infty, 2) \cup (2, \infty) = \mathbb{R} - \{2\}$.

Draw it, do not argue it. Sketch both intervals on one number line with open and closed dots. Every endpoint question answers itself once the picture is there.

28 SOLUTION SETS OF INEQUALITIES
E·28

A set written in builder form is usually the solution set of an inequality, so solve first and describe second.

KEY

$$\{x \in \mathbb{R} : x^2 - 5x + 6 \le 0\} = [2, 3]$$

i. Bring everything to one side and factorise.
ii. Mark the roots on a number line and read off the sign in each part.
iii. Include a root when the inequality is $\le$ or $\ge$, exclude it when it is strict.
iv. A root of a denominator is always excluded, whatever the inequality sign.

Never cross multiply by a variable. For $\frac{x-1}{x-2} \ge 0$ the sign of $x - 2$ is unknown, so multiplying by it can flip the inequality without warning. Move everything to one side and use the sign line instead.

29 SETS DESCRIBED WITH $|x|$ AND $[x]$
E·29

- $\{x : |x| < a\} = (-a, a)$ and $\{x : |x| > a\} = (-\infty, -a) \cup (a, \infty)$, for $a > 0$.
- $\{x : |x - c| \le r\} = [c - r,\ c + r]$: the points within r of c.
- $\{x : [x] = k\} = [k, k + 1)$ for $k \in \mathbb{Z}$, where $[x]$ is the greatest integer function; for any other k the set is empty.
- $\{x : \{x\} = 0\} = \mathbb{Z}$, where $\{x\} = x - [x]$ is the fractional part.
- $\{x \in \mathbb{Z} : |x| \le 3\}$ has 7 elements, not infinitely many — the universe was $\mathbb{Z}$.

The universe changes the answer. $\{x : x^2 < 5\}$ is an interval in $\mathbb{R}$ but the finite set $\{-2, -1, 0, 1, 2\}$ in $\mathbb{Z}$. Read the part before the colon before doing any algebra.

30 QUICK FACTS AND COMMON TRAPS
E·30

i. $\emptyset \neq \{\emptyset\}$: the first has no elements, the second has one.
ii. $\in$ relates an element to a set, $\subseteq$ relates a set to a set. Both can be true at once — $\emptyset \in \{\emptyset\}$ and $\emptyset \subseteq \{\emptyset\}$ — but they never mean the same thing.
iii. $n(P(A)) = 2^{n(A)}$, so a power set of a 5 element set has 32 elements, not 10.
iv. $(A \cup B)' = A' \cap B'$: the operation flips when the prime goes inside.
v. $A - B$ and $B - A$ need not be equal, and neither is $A \triangle B$ unless the other one is empty.
vi. $n(A \cup B) = n(A) + n(B)$ only when the sets are disjoint.
vii. In three set counting, exactly one carries $+3S_3$ and exactly two carries $-3S_3$.
viii. Equivalent sets need only match in size; equal sets must match element by element.
ix. Relations, equivalence classes and $A \times B$ are on the Relations and Functions sheet.