

[OLYMPIAD MATHEMATICS]

IOQM · PRE-RMO

2027

IOQM *Syllabus*

CLASS 8 TO 12

OLYMPIAD

UNIT WISE

IN THIS SYLLABUS

PAPER	IOQM MATHEMATICS
UNITS	NINETEEN, IN FOUR AREAS
SCOPE	FULL OFFICIAL SYLLABUS
DETAIL FROM	IOQM 2026 · OLYMPIAD ARCHIVE
ALSO CARRIES	FORMULA BANKS · PRO TIPS
APPLIES TO	IOQM 2027
PAIRED WITH	THE IOQM HANDBOOK

IOQM Mathematics Syllabus

Algebra · Geometry · Combinatorics · Number Theory

FOR IOQM 2027 | EDITION 1 | 2026

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[IOQM]

How to read this syllabus

This syllabus is for IOQM, the Indian Olympiad Qualifier in Mathematics. IOQM is the first exam of the olympiad route. Students who clear it write RMO, then INMO, and the best of them go on to the IMO. So whatever you study for IOQM will help you at every later stage.

We have started from the official syllabus published by HBCSE. It is given in full on the next page, and every topic in it is covered in the units of this syllabus.

The official list is very short, only a few lines for each area. So we have added the detail under every unit, and also some topics that the official list does not mention. All of this comes from real questions: the 30 questions of IOQM 2026 and our own olympiad question archive. If a topic came in IOQM 2026, the question number is written next to it.

There are 19 units in four areas: algebra, geometry, combinatorics and number theory. Ten units are marked HIGH WEIGHTAGE. Questions come from these units most often, going by the 2026 paper and our archive. This does not mean they are harder, and it does not mean you can leave the other units.

After each unit there are two boxes. The first has the formulas and results you should remember. The second has tips on how to start a question of that kind. Read the topic list first; the boxes are only there to help.

Use this syllabus together with the IOQM Handbook, where every unit is taught with solved examples and exercises.

OFFICIAL SYLLABUS · HBCSE · MATHEMATICAL OLYMPIAD

Scope: Pre-degree college mathematics. The syllabus does not include calculus and statistics. HBCSE adds that problems may also come from topics not stated explicitly.

Algebra: Inequalities; progressions (A.P., G.P., H.P.); theory of indices; system of linear equations; theory of equations; binomial theorem and properties of binomial coefficients; complex numbers; polynomials in one and two variables; functional equations; sequences.

Plane Geometry: Triangles; quadrilaterals; circles and their properties; standard Euclidean constructions; concurrency and collinearity (theorems of Ceva and Menelaus); basic trigonometric identities; compound angles; multiple and submultiple angles; general solutions; sine rule; cosine rule; properties of triangles and polygons; coordinate geometry (straight line, circle, conics, 3-D geometry); vectors.

Combinatorics: Basic enumeration; pigeonhole principle and its applications; recursion; elementary graph theory.

Number Theory: Divisibility theory in the integers (the division algorithm, the greatest common divisor, the Euclidean algorithm, the Diophantine equation $ax + by = c$); fundamental theorem of arithmetic; basic properties of congruence; linear congruences; Chinese remainder theorem; Fermat's little theorem; Wilson's theorem; Euler's phi function and Euler's generalisation of Fermat's theorem; Pythagorean triples (definition and properties); Diophantine equations.

Also Named by HBCSE: Arithmetic of integers; quadratic equations and expressions; factorisation of polynomials; permutations and combinations; elementary combinatorics; probability theory; finite series.

THE PAPER · HOW IOQM IS SET

- 30 questions in 3 hours, for 100 marks.
- Q1 to Q10 carry 2 marks each, Q11 to Q20 carry 3 marks each, and Q21 to Q30 carry 5 marks each.
- Every answer is a whole number from 00 to 99. There are no options and no negative marking.
- No calculus and no statistics. Everything is school mathematics, but almost every question needs an idea as well as a method.

[AREA 1 OF 4]

Algebra *units 01 to 05*

UNIT 01 Identities, Indices and Surds

HIGH WEIGHTAGE

- **Theory of Indices:** Laws of exponents for integral and rational powers; Comparing large powers by bringing them to a common exponent (2^{300} against 3^{200}); **Exponential equations that reduce to an algebraic identity**, such as $9^x + 4^x + 1 = 6^x + 3^x + 2^x$.
- **Square Identities:** $(a \pm b)^2$ and $a^2 - b^2 = (a - b)(a + b)$; **Completing the square;** $a^2 + b^2 + c^2 - ab - bc - ca = \frac{1}{2} [(a - b)^2 + (b - c)^2 + (c - a)^2]$; **A sum of real squares is zero only when every square is zero.**
- **Classic Factorisations: Sophie Germain:** $a^4 + 4b^4 = (a^2 + 2ab + 2b^2)(a^2 - 2ab + 2b^2)$; $x^4 + x^2 + 1 = (x^2 + x + 1)(x^2 - x + 1)$; Product of four consecutive integers plus one: $n(n + 1)(n + 2)(n + 3) + 1 = (n^2 + 3n + 1)^2$.
- **Cubic Identities:** $a^3 \pm b^3$; $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$; **If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$.**
- **Algebraic Manipulation:** Symmetric expressions written through $a + b$ and ab ; Systems solved by adding, subtracting or combining whole equations; **Cyclic systems:** subtract the equations in pairs and factorise (IOQM 2026 Q21).
- **Surds:** Rationalising denominators; Nested radicals $\sqrt{a \pm 2\sqrt{b}} = \sqrt{x} \pm \sqrt{y}$ when $x + y = a$ and $xy = b$ (with $x > y$ for the minus sign); Products such as $(\sqrt{5} + \sqrt{6} + \sqrt{7})(-\sqrt{5} + \sqrt{6} + \sqrt{7})(\sqrt{5} - \sqrt{6} + \sqrt{7})(\sqrt{5} + \sqrt{6} - \sqrt{7})$.

HIGH-PRIORITY · IDENTITIES THE PAPER RETURNS TO

Formula Bank: Each of these is used in the archive or in the IOQM 2026 paper.

- $n^4 + 4 = (n^2 + 2n + 2)(n^2 - 2n + 2)$, so for a positive integer n , $n^4 + 4$ is prime only when $n = 1$.
- $\sqrt{n(n + 1)(n + 2)(n + 3) + 1} = n^2 + 3n + 1$; for example $\sqrt{31 \cdot 30 \cdot 29 \cdot 28 + 1} = 869$.
- $a^4 + b^4 + (a + b)^4 = 2(a^2 + ab + b^2)^2$ and $a^2 + b^2 + (a + b)^2 = 2(a^2 + ab + b^2)$.
- $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$, used again in IOQM 2026 Q24 with $a = p^2$, $b = q^2$.
- $a^4 + a^2b^2 + b^4 = (a^2 + ab + b^2)(a^2 - ab + b^2)$.

PRO-TIP · SPOTTING THE IDENTITY

Exponential Equations: Put $a = 3^x$ and $b = 2^x$. Then $9^x + 4^x + 1 = 6^x + 3^x + 2^x$ becomes

$$a^2 + b^2 + 1 - ab - a - b = \frac{1}{2} \left[(a-b)^2 + (a-1)^2 + (b-1)^2 \right] = 0,$$

so $a = b = 1$ and $x = 0$. Whenever an equation has as many terms on each side and looks symmetric, test the sum-of-squares form first.

Find all real x, y : A single equation in two real unknowns usually means a sum of squares equal to zero. For RMO 2014, $x^2 + 2y^2 + \frac{1}{2} \leq x(2y + 1)$ is the same as $\left(x - y - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 \leq 0$.

UNIT 02 Polynomials and Theory of Equations

- **Polynomials:** Degree, coefficients and equality of polynomials (**compare coefficients**); Division algorithm $P(x) = D(x)Q(x) + R(x)$ with $\deg R < \deg D$; Polynomials in one and two variables.
- **Remainder and Factor Theorems:** Remainder on division by $x - a$ is $P(a)$; $P(a) = 0$ **if and only if** $(x - a)$ divides $P(x)$; Remainder on division by a quadratic with two distinct roots, found from the values at those roots.
- **Factorisation of Polynomials:** Grouping, splitting the middle term and the factor theorem; Symmetric and cyclic polynomials such as $(a - b)(b - c)(c - a)$; Factorising $x^n - 1$ and $x^n + 1$.
- **Quadratic Equations and Expressions:** Discriminant and nature of roots; Sign of $ax^2 + bx + c$ on the real line; Greatest and least values by completing the square; **Common roots of two quadratics.**
- **Theory of Equations:** Vieta's relations for quadratics, cubics and quartics; Symmetric functions of the roots; Forming an equation with given roots; Roots in A.P. or G.P., or roots that are side lengths.
- **Integer and Rational Roots:** Rational root test; Integer roots divide the constant term; For integer coefficients, $a - b$ divides $P(a) - P(b)$; Repeated roots, where $(x - a)^2$ divides $P(x)$.
- **Building a New Polynomial: Subtract the pattern:** if $P(1) = 10, P(2) = 20, P(3) = 30$, then $P(x) - 10x$ has roots 1, 2, 3, and for a monic quartic it equals $(x - 1)(x - 2)(x - 3)(x - t)$; **Composition and degree count** for equations such as $P(x^2) = P(P(x))$ (IOQM 2026 Q11).
- **System of Linear Equations:** Two and three unknowns; Elimination; Unique, no or infinitely many solutions; **Integer solutions of a system with fewer equations than unknowns** (combine the equations, then use bounds).

HIGH-PRIORITY · VIETA AND POWER SUMS

Vieta for a Cubic: If $x^3 + px^2 + qx + r = 0$ has roots α, β, γ , then

$$\alpha + \beta + \gamma = -p, \quad \alpha\beta + \beta\gamma + \gamma\alpha = q, \quad \alpha\beta\gamma = -r.$$

Power Sums: With e_1, e_2, e_3 the three elementary symmetric sums,

- $\alpha^2 + \beta^2 + \gamma^2 = e_1^2 - 2e_2$
- $\alpha^3 + \beta^3 + \gamma^3 = e_1^3 - 3e_1e_2 + 3e_3$
- For the roots of $ax^2 + bx + c = 0$: $aS_n + bS_{n-1} + cS_{n-2} = 0$, where $S_n = \alpha^n + \beta^n$.

Subtract the Pattern: In ARML 1984, P is a monic quartic, so $P(x) - 10x = (x-1)(x-2)(x-3)(x-t)$; the unknown root t cancels in $P(12) + P(-8)$, giving $\frac{P(12)+P(-8)}{10} = 1984$.

PRO-TIP · READING A POLYNOMIAL QUESTION

Degrees First: In a functional equation between polynomials, compare degrees before anything else. $P(x^2) = P(P(x))$ gives $2n = n^2$, so $n = 2$ for a non-constant P ; then compare the coefficients of x^4 , x^3 and x^2 in turn.

Common Root: If two quadratics share a root α , subtract a suitable multiple of one from the other to remove α^2 ; α then comes out of a linear equation.

Two Roots Given in Combination: When a quartic has two roots with a known sum or product, write it as a product of two quadratics and compare coefficients.

UNIT 03 Complex Numbers and the Binomial Theorem

- **Complex Numbers:** Algebra of $a + ib$; Conjugate and modulus, with $|z|^2 = z\bar{z}$; Argand plane; Polar form $r(\cos \theta + i \sin \theta)$; $|z_1 z_2| = |z_1| |z_2|$.
- **De Moivre and Roots of Unity:** $(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta$; n th roots of unity as vertices of a regular n -gon; $1 + \omega + \omega^2 = 0$ and $\omega^3 = 1$; Sum and product of the roots.
- **Complex Numbers in Algebra:** Non-real roots of a real polynomial come in conjugate pairs; Factorising $x^n - 1$ over the complex numbers; Systems of equations over $\mathbb{C}$ (IOQM 2026 Q21 allows complex x, y, z).
- **Binomial Theorem:** Expansion for a positive integral index; General term $T_{r+1} = {}^nC_r a^{n-r} b^r$; Middle term; Term independent of x .
- **Properties of Binomial Coefficients:** $\sum_r {}^nC_r = 2^n$ and the alternating sum 0; **Pascal's rule** ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$; **Vandermonde:** $\sum_k {}^mC_k {}^nC_{r-k} = {}^{m+n}C_r$; **Hockey stick:** $\sum_{i=r}^n {}^iC_r = {}^{n+1}C_{r+1}$.
- **Applications:** Remainder of a^n by writing $a = mk \pm 1$; Sums of coefficients from $P(1)$ and $P(-1)$; **Pascal-type triangles**, where each entry is the sum of the two below it.

HIGH-PRIORITY · COEFFICIENT SUMS AND THE NUMBER TRIANGLE

Filters: For $P(x) = a_0 + a_1x + a_2x^2 + \dots$,

- $a_0 + a_1 + a_2 + \dots = P(1)$
- $a_0 + a_2 + a_4 + \dots = \frac{1}{2} [P(1) + P(-1)]$
- $a_0 + a_3 + a_6 + \dots = \frac{1}{3} [P(1) + P(\omega) + P(\omega^2)]$

Number Triangle: If each row is formed by adding neighbours, the top entry built on $x_1, x_2, \dots, x_n$ is

$$\sum_{k=1}^n {}^{n-1}C_{k-1} x_k.$$

For six numbers the weights are 1, 5, 10, 10, 5, 1, so modulo 5 only $x_1 + x_6$ matters (AIMO 2022).

PRO-TIP · WHEN TO BRING IN ω

Remainders: To find the remainder of x^{2026} on division by $x^2 + x + 1$, put $x = \omega$: since $\omega^3 = 1$, $\omega^{2026} = \omega$, and the remainder is x .

Factorising with ω : $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a + b\omega + c\omega^2)(a + b\omega^2 + c\omega)$ and $x^2 + y^2 = (x + iy)(x - iy)$ turn sums into products.

UNIT 04 Sequences, Progressions and Series

HIGH WEIGHTAGE

- **Progressions:** A.P., G.P. and H.P.: n th term and sum of n terms; Arithmetic, geometric and harmonic means; Three terms in A.P. taken as $a - d, a, a + d$.
- **Integer Progressions:** A G.P. of integers with ratio p/q in lowest terms has terms kq^2, kpq, kp^2 ; Counting such progressions with a fixed last term (archive: $c = 8000$).
- **Finite Series:** $\sum k, \sum k^2, \sum k^3$; **Telescoping** with $\frac{1}{k(k+1)} = \frac{1}{k} - \frac{1}{k+1}$ and its product forms; Sum of the integers from $m + 1$ to n is $\frac{(n-m)(n+m+1)}{2}$.
- **Recurrences:** $x_{n+1} = ax_n + b$; Linear recurrences $a_{n+2} = pa_{n+1} + qa_n$; **Fibonacci-type sequences**, $a_{n+1} = a_n + a_{n-1}$ (IOQM 2026 Q28).
- **Periodic Sequences:** Write out the first eight terms and look for a cycle; $a_{n-1}a_{n+1} = a_n$ has period 6; Reducing the index of a far term by the period.
- **Invariants of a Recurrence:** Rearranging a relation until a product or a difference stays constant (IOQM 2026 Q16 reduces to $a_{n+1}a_{n+3} = a_na_{n+2}$, which makes the sequence periodic with period 4).
- **Sums of Powers:** $s_n = ax^n + by^n$ satisfies $s_{n+1} = (x + y)s_n - xy s_{n-1}$ (archive: AIME 1990, where $s_1, \dots, s_4 = 3, 7, 16, 42$ give $s_5 = 20$).

HIGH-PRIORITY · SERIES RESULTS

Formula Bank:

- $\sum_{k=1}^n k = \frac{n(n+1)}{2}, \quad \sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{k=1}^n k^3 = \left(\frac{n(n+1)}{2}\right)^2$
- $\sum_{k=1}^n \frac{1}{k(k+1)(k+2)} = \frac{1}{4} - \frac{1}{2(n+1)(n+2)}$
- For positive numbers, A.M. $\geq$ G.M. $\geq$ H.M.; for two numbers, $G^2 = AH$.

Fibonacci Form: With $F_1 = F_2 = 1$, any sequence with $a_{n+1} = a_n + a_{n-1}$ satisfies

$$a_n = F_{n-2} a_1 + F_{n-1} a_2, \quad \text{so} \quad a_{10} = 21a_1 + 34a_2.$$

PRO-TIP · TERMS BEFORE FORMULAS

Non-linear Recurrences: When the rule is not linear, compute a_1 to a_8 before trying anything clever. A cycle, a constant product or a constant difference usually appears.

Large Index: Questions ask for a_{2026} or for a sum up to 2026 because the period makes them easy. Divide the index by the period and keep the remainder; a remainder 0 means the last term of the cycle.

UNIT 05 Inequalities and Functional Equations

- **Basic Inequalities:** Rules for multiplying by negatives and taking reciprocals; Squares are non-negative; **Triangle inequality** $|a+b| \leq |a| + |b|$; Absolute value as a distance on the number line.
- **Means: A.M.-G.M.** for n positive numbers, with equality exactly when all are equal; The chain Q.M. $\geq$ A.M. $\geq$ G.M. $\geq$ H.M.; $x + \frac{1}{x} \geq 2$ for $x > 0$.
- **Cauchy-Schwarz:** $(\sum a_i^2)(\sum b_i^2) \geq (\sum a_i b_i)^2$; **Titu's form** $\sum \frac{a_i^2}{b_i} \geq \frac{(\sum a_i)^2}{\sum b_i}$ for $b_i > 0$.
- **Greatest and Least Values:** Quadratic expressions; Products with a fixed sum and sums with a fixed product; **Least sum of squares of integers under a condition**, where parity lifts the bound (IOQM 2026 Q3).
- **Functional Equations:** Substituting special values ($x = 0, y = x, y = 0, x = 1$); **Cyclic substitutions** $x \rightarrow \frac{1}{1-x} \rightarrow \frac{x-1}{x} \rightarrow x$, which return after three rounds; Solving the resulting linear system for $f(x)$.
- **Finding All Functions:** Guessing the solution, showing no other function works, and checking it in the original equation; Functional equations for polynomials (Unit 02).

HIGH-PRIORITY · THE TWO INEQUALITIES TO KNOW

A.M.-G.M.: For $a_1, \dots, a_n > 0$,

$$\frac{a_1 + a_2 + \dots + a_n}{n} \geq \sqrt[n]{a_1 a_2 \dots a_n},$$

with equality if and only if $a_1 = a_2 = \dots = a_n$. Always write down the equality case: it tells you whether the bound is reached.

Integers Change the Bound: For 49 non-zero integers with sum 0, each square is at least 1, but they cannot all be ± 1 , because an odd number of ± 1 's has an odd sum. The least possible sum of squares is therefore larger than 49 (IOQM 2026 Q3).

PRO-TIP · FUNCTIONAL EQUATIONS IN IOQM

A Value, Not a Proof: IOQM asks for a number such as $12f(12)$ or $f\left(\frac{1}{2}\right)$. Substitute the values that make the arguments repeat, get two or three linear equations in the unknown values, and solve them.

The Three-Cycle: x , $\frac{1}{1-x}$ and $\frac{x-1}{x}$ go round in a cycle of three, so an equation in $f(x)$ and $f\left(\frac{1}{1-x}\right)$ needs exactly three substitutions (archive: AMO 2003, Singapore 2011).

[AREA 2 OF 4]

Geometry *units 06 to 10*

UNIT 06 Triangles, Polygons and Angle Chasing

HIGH WEIGHTAGE

- **Angles:** Parallel lines and transversals; Angle sum 180° ; Exterior angle theorem; **Isosceles triangle: equal sides face equal angles**, and the converse.
- **Congruence:** SSS, SAS, ASA, RHS; Building a congruent copy to carry an angle or a length to a better place.
- **Triangle Inequality and Side Order:** The larger side faces the larger angle; Range of the third side; Triangles with integer sides.
- **Special Triangles:** Equilateral, 30° - 60° - 90° and 45° - 45° - 90° triangles; **The median to the hypotenuse is half the hypotenuse**; Midpoint theorem.
- **Pythagoras:** Pythagoras' theorem and its converse; Acute or obtuse test by comparing c^2 with $a^2 + b^2$; Distances between points of a square grid.
- **Quadrilaterals:** Parallelogram, rectangle, rhombus, square, trapezium and kite with their properties; Angle sum 360° (IOQM 2026 Q8 is a trapezium).
- **Polygons:** Interior angle sum $(n - 2) \cdot 180^\circ$; Exterior angles add to 360° ; Regular polygons; **Equiangular hexagons** and polygons cut from an equilateral triangle.
- **Euclidean Constructions:** Bisecting a segment and an angle; Perpendiculars and parallels; Constructing triangles from given parts; **Auxiliary constructions:** an equilateral triangle on a side, a reflection, a produced side.
- **Angle Chasing:** Writing every angle in terms of one unknown; Equal angles from isosceles triangles and parallel lines (IOQM 2026 Q8, Q19).

HIGH-PRIORITY · FACTS IN NEARLY EVERY FIGURE

Formula Bank:

- Exterior angle of a triangle = sum of the two opposite interior angles.
- In a right triangle the midpoint of the hypotenuse is equidistant from all three vertices.
- Sides of the 30° - 60° - 90° triangle are $1 : \sqrt{3} : 2$; of the 45° - 45° - 90° triangle, $1 : 1 : \sqrt{2}$.
- Interior angle of a regular n -gon is $\frac{(n - 2) \cdot 180^\circ}{n}$.
- An equiangular hexagon with sides $a_1, \dots, a_6$ in order satisfies $a_1 - a_4 = a_5 - a_2 = a_3 - a_6$.

PRO-TIP · WHEN ANGLE CHASING STOPS

Look for the Missing Isosceles Triangle: If the angles will not close, construct one: an equilateral triangle on a known side, or a point that makes two lengths equal. Many angle questions in the archive open this way.

Draw to Scale: Use a ruler and protractor on a large, accurate figure. IOQM answers are whole numbers, so a measured 29.8° is a strong sign that the answer is 30, and a wrong guess of the configuration shows up at once.

UNIT 07 Similarity, Lengths and Areas

- **Similarity:** AA, SAS and SSS similarity; **Areas of similar figures are in the square of the ratio of sides;** Basic proportionality theorem and its converse.
- **Angle Bisector Theorem:** $BD : DC = AB : AC$ for the internal bisector, and the external form; Length of the bisector.
- **Right Triangles:** **Altitude to the hypotenuse**, cutting c into p next to leg a and q next to leg b : $h^2 = pq$, $a^2 = pc$, $b^2 = qc$; Altitude = (product of legs)/(hypotenuse) (IOQM 2026 Q24).
- **Medians and Cevians:** Centroid divides each median in the ratio 2 : 1; **Apollonius:** $AB^2 + AC^2 = 2AD^2 + 2BD^2$ for the median AD ; **Stewart:** $b^2m + c^2n = a(d^2 + mn)$.
- **Area Formulas:** $\frac{1}{2}bh$, $\frac{1}{2}ab \sin C$, **Heron's formula**, $\Delta = rs = \frac{abc}{4R}$; Area of a quadrilateral from its diagonals; Altitudes $h_a = \frac{2\Delta}{a}$.
- **Ratios of Areas:** Triangles with the same height; Triangles sharing an angle: $\frac{[AEF]}{[ABC]} = \frac{AE \cdot AF}{AB \cdot AC}$; Regions cut off by parallel lines; **Triangles formed by the diagonals of a trapezium.**
- **Concurrency and Collinearity:** Ceva's theorem for concurrent cevians; Menelaus' theorem for collinear points; **Mass points** for ratios along cevians (archive: RMO 2012).

HIGH-PRIORITY · LENGTH AND AREA RESULTS

Formula Bank: In triangle ABC with $s = \frac{a+b+c}{2}$,

- Heron: $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$; also $\Delta = rs = \frac{abc}{4R}$.
- Stewart: if D on BC has $BD = m$, $DC = n$, $AD = d$, then $b^2m + c^2n = a(d^2 + mn)$.
- Bisector from A : $AD^2 = bc \left[1 - \frac{a^2}{(b+c)^2} \right]$.
- Trapezium $ABCD$ ($AB \parallel CD$) with diagonals meeting at O : $[AOD] = [BOC]$ and $[AOD]^2 = [AOB][COD]$.
- Since $h_a = \frac{2\Delta}{a}$, the numbers $\frac{1}{h_a}, \frac{1}{h_b}, \frac{1}{h_c}$ satisfy the triangle inequality.

PRO-TIP · AREAS BEFORE LENGTHS

Areas as Fractions: When a figure gives areas, write every unknown area as a fraction of one known area using shared heights; a ratio of lengths on one line is a ratio of areas.

Coordinates Are Allowed: When two cevians cross and both ratios are needed, putting $B = (0,0)$ and $C = (1,0)$ is often quicker than Menelaus. The IITianForum solution of IOQM 2026 Q24 places the rectangle on axes.

UNIT 08 Circles

- **Chords and Arcs:** Equal chords subtend equal angles at the centre; The perpendicular from the centre bisects a chord; Arcs and central angles.
- **Inscribed Angles:** An inscribed angle is half the central angle on the same arc; Angles in the same segment are equal; The angle in a semicircle is 90° .
- **Cyclic Quadrilaterals:** Opposite angles add to 180° ; Exterior angle equals the interior opposite angle; Tests for four points to be concyclic.
- **Tangents:** Tangent perpendicular to the radius; Equal tangents from an external point; **Alternate segment theorem**; Tangential quadrilateral: $AB + CD = BC + DA$.
- **Power of a Point:** Intersecting chords $PA \cdot PB = PC \cdot PD$; Secant and tangent $PT^2 = PA \cdot PB$; Radical axis of two circles.
- **Two or More Circles:** Touching circles: distance between centres $r_1 + r_2$ or $|r_1 - r_2|$; Lengths of common tangents; Circles packed in a square, a rhombus or an equilateral triangle.
- **Circles of a Triangle:** Circumcircle with $a = 2R \sin A$; Incircle with tangent lengths $s - a, s - b, s - c$; Excircles; **Arc midpoints:** the perpendicular bisector of BC meets the circumcircle at the midpoints of the two arcs BC (IOQM 2026 Q5), and the bisector of angle A passes through one of them.
- **Ptolemy and Triangle Centres:** Ptolemy's theorem $AC \cdot BD = AB \cdot CD + AD \cdot BC$ for a cyclic quadrilateral; Centroid, incentre, circumcentre and orthocentre; Distances from the incentre to the vertices (archive: $AI \cdot BI \cdot CI$ in the 3-4-5 triangle).

HIGH-PRIORITY · CIRCLE RESULTS

Formula Bank:

- Tangent from a point at distance d from the centre: length $\sqrt{d^2 - r^2}$.
- Two circles with centres d apart: direct common tangent $\sqrt{d^2 - (r_1 - r_2)^2}$, transverse common tangent $\sqrt{d^2 - (r_1 + r_2)^2}$.
- Incircle of a right triangle with legs a, b and hypotenuse c : $r = \frac{a + b - c}{2}$.
- Cyclic quadrilateral with perpendicular diagonals meeting at E : $EA^2 + EB^2 + EC^2 + ED^2 = 4R^2$ (RMO 1992).

PRO-TIP · FIND THE HIDDEN CIRCLE

Equal Angles on One Segment: If two equal angles stand on the same segment and lie on the same side of it, the four points are concyclic. Draw that circle first; the inscribed angle theorem then gives the rest.

Arc Midpoints: The midpoint M of arc BC (not containing A) is equidistant from B, C and the incentre I . Perpendicular bisectors and angle bisectors both pass through such points.

UNIT 09 Trigonometry

HIGH WEIGHTAGE

- **Basic Identities:** Ratios of an acute angle; $\sin^2 \theta + \cos^2 \theta = 1$, $1 + \tan^2 \theta = \sec^2 \theta$, $1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$; Values at $0^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ$.
- **Compound Angles:** $\sin(A \pm B)$, $\cos(A \pm B)$, $\tan(A \pm B)$; Exact values such as $\sin 15^\circ$ and $\tan 75^\circ$ (IOQM 2026 Q13); Sum-to-product and product-to-sum.
- **Multiple and Submultiple Angles:** $\sin 2A$, $\cos 2A$, $\tan 2A$; $\sin 3A$ and $\cos 3A$; Half-angle formulas; $\sin 18^\circ = \frac{\sqrt{5}-1}{4}$.
- **General Solutions:** $\sin \theta = \sin \alpha$, $\cos \theta = \cos \alpha$, $\tan \theta = \tan \alpha$; Equations that become quadratics in one ratio.
- **Sine and Cosine Rules:** $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$; $c^2 = a^2 + b^2 - 2ab \cos C$; **Finding an unknown angle in a figure through the sine rule** (IOQM 2026 Q30) and the cosine rule (IOQM 2026 Q27).
- **Properties of Triangles:** $\Delta = \frac{1}{2}ab \sin C$; Identities when $A + B + C = 180^\circ$, such as $\tan A + \tan B + \tan C = \tan A \tan B \tan C$ (no angle 90°); Projection formula $a = b \cos C + c \cos B$.
- **Trigonometric Expressions:** Setting $t = \sin \theta + \cos \theta$, so that $\sin 2\theta = t^2 - 1$; Expressions in all six ratios reduced to $\sin \theta$ and $\cos \theta$.

HIGH-PRIORITY · VALUES AND THE TWO RULES

Exact Values:

- $\sin 15^\circ = \frac{\sqrt{6} - \sqrt{2}}{4}$, $\cos 15^\circ = \frac{\sqrt{6} + \sqrt{2}}{4}$, $\tan 15^\circ = 2 - \sqrt{3}$, $\tan 75^\circ = 2 + \sqrt{3}$
- $\sin 18^\circ = \frac{\sqrt{5} - 1}{4}$, $\cos 36^\circ = \frac{\sqrt{5} + 1}{4}$

The Two Rules:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R, \quad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

PRO-TIP · TRIGONOMETRY INSIDE GEOMETRY

One Unknown Angle: When angle chasing gives every angle except one, call it x . Apply the sine rule in two triangles that share a side and equate the two expressions for that side. The result is an equation of the form $\sin(\cdot)/\sin(\cdot) = \sin(\cdot)/\sin(\cdot)$, and the answer is a whole number of degrees.

Where It Was Used: IOQM 2026 used trigonometry in three geometry questions: Q13, Q27 and Q30.

UNIT 10 Coordinate Geometry and Vectors

- **Points and Lines:** Distance and section formulas; Area of a triangle from coordinates; Slope; Forms of the equation of a line; Parallel and perpendicular lines; Distance of a point from a line.
- **Circles:** $(x - h)^2 + (y - k)^2 = r^2$ and the general form; A line and a circle; Condition for tangency.
- **Conics:** Standard forms of the parabola, ellipse and hyperbola; Focus, directrix and eccentricity.
- **Three-Dimensional Geometry:** Coordinates in space; Distance and section formulas; Direction ratios.
- **Vectors:** Addition and scalar multiples; Position vectors; Section formula in vector form; Dot product and perpendicularity; Vector proofs of concurrency.
- **Coordinates as a Method: Placing a figure on axes:** rectangles, right angles, squares and regular hexagons; **Lattice points** and distances on a grid (archive: integer distances in a 4×4 grid, circles through exactly two grid points).

HIGH-PRIORITY · COORDINATE FORMULAS

Formula Bank:

- Distance of (x_0, y_0) from $ax + by + c = 0$: $\frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$.
- Area of the triangle with vertices (x_i, y_i) : $\frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$.
- Point dividing AB in the ratio $m : n$: $\left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n} \right)$; centroid $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$.

PRO-TIP · WHEN TO SWITCH TO COORDINATES

A Method, Not a Topic: In IOQM 2026 and in the archive, coordinates and vectors appear as a method of solving geometry questions rather than as questions of their own. Choose axes along the right angles of the figure and put a vertex at the origin.

[AREA 3 OF 4]

Combinatorics *units 11 to 14*

UNIT 11 Permutations, Combinations and Bijections

HIGH WEIGHTAGE

- **Counting Principles:** Addition and multiplication principles; **Complementary counting;** Splitting into cases that do not overlap.
- **Permutations:** ${}^n P_r$; Arrangements with repeated objects $\frac{n!}{p!q!\dots}$; Circular arrangements $(n-1)!$; Objects kept together or kept apart.
- **Combinations:** ${}^n C_r$ and Pascal's triangle; Selections with restrictions; Subsets of a set and subsets with a condition.
- **Numbers with Digit Conditions:** Counting numbers by their digits, with no leading zero; Largest or smallest numbers built from distinct digits (IOQM 2026 Q4); **Arrangements that spell the same string** counted once (IOQM 2026 Q22).
- **Distributions: Stars and bars:** $x_1 + \dots + x_k = n$ has $\binom{n+k-1}{k-1}$ non-negative solutions; Upper bounds handled by inclusion and exclusion; Distinct objects into boxes.
- **Inclusion and Exclusion:** $|A \cup B \cup C|$ for three sets; Arrangements with forbidden neighbours; **Derangements** D_n .
- **Bijections and Double Counting:** Counting one set in two ways; Matching each object with a simpler one; Chessboard colours read from the parity of $i + j$ (IOQM 2026 Q1).
- **Order and Absolute Values:** For $x \leq y \leq z$, $|x - y| + |y - z| + |z - x| = 2(z - x)$, which turns a condition on three numbers into one on the largest and smallest (IOQM 2026 Q17).
- **Counting in Grids:** Rectangles and squares in an $m \times n$ grid; Matchsticks needed for an $a \times b$ grid of squares, $2ab + a + b$.

HIGH-PRIORITY · COUNTING FORMULAS

Formula Bank:

- Stars and bars: $\binom{n+k-1}{k-1}$ non-negative solutions, $\binom{n-1}{k-1}$ positive solutions.
- Derangements: $D_n = n! \left(1 - \frac{1}{1!} + \frac{1}{2!} - \dots + \frac{(-1)^n}{n!}\right)$; $D_2, \dots, D_6 = 1, 2, 9, 44, 265$.
- Rectangles in an $m \times n$ grid of unit squares: $\binom{m+1}{2} \binom{n+1}{2}$; squares in an $n \times n$ grid: $1^2 + 2^2 + \dots + n^2$.
- Circular arrangements: $(n-1)!$; if the circle may be turned over, $\frac{(n-1)!}{2}$.

PRO-TIP · HOW IOQM ASKS FOR A COUNT

The 00 to 99 Answer: A large count is asked as its remainder on division by 100 (IOQM 2026 Q18, Q23) or as the sum of its digits (Q4, Q22). Find the exact count first and reduce only at the end.

Repeated Strings: When different arrangements can spell the same string, as the blocks 1, 0 against the block 10 in IOQM 2026 Q22, count the arrangements, then remove the repeats by inclusion and exclusion.

UNIT 12 Recursion and Casework

- **Setting Up a Recursion:** Classify by the first or last object; Tilings with pieces of length 1 and 2 give $a_n = a_{n-1} + a_{n-2}$ (archive: ragas of 11 beats); Strings with rules on neighbouring digits.
- **Classic Recursions:** Fibonacci; Binary strings with no two adjacent 1s; Stair climbing; Derangements $D_n = (n-1)(D_{n-1} + D_{n-2})$.
- **Counting Stage by Stage:** A table of states (last digit, current vertex) updated one position at a time; Walks on a polygon (archive: ARML 2004).
- **Lattice Paths:** $\binom{m+n}{m}$ shortest paths; Paths that never cross the diagonal and **Catalan numbers** $C_n = \frac{1}{n+1} \binom{2n}{n}$; Reflection.
- **Casework:** Splitting by the largest part or by the position of a special digit; **Cases that are disjoint and cover everything** (IOQM 2026 Q7: digits from $\{1, 2, 4, 8\}$, a 1 only after an 8).
- **Linear Recurrences:** Characteristic equation of $a_n = pa_{n-1} + qa_{n-2}$; Closed forms; Checking a closed form against the first terms.
- **Counting Processes:** Tournaments and games counted as lattice paths: a relay between two teams of n players has $\binom{2n}{n}$ possible processes (archive: China 1988).

HIGH-PRIORITY · SEQUENCES WORTH KNOWING BY SIGHT

Formula Bank:

- Tilings of a $1 \times n$ strip by 1×1 and 1×2 pieces: F_{n+1} , that is 1, 2, 3, 5, 8, 13, ...
- Binary strings of length n with no two adjacent 1s: F_{n+2} .
- Catalan numbers: 1, 1, 2, 5, 14, 42, 132, 429, 1430.
- Shortest grid paths from $(0, 0)$ to (m, n) : $\binom{m+n}{m}$.

PRO-TIP · SMALL CASES NAME THE SEQUENCE

Count by Hand First: Count $n = 1, 2, 3, 4$ directly before writing a formula. The numbers often name the sequence at once: Fibonacci, Catalan, powers of 2.

When the Rule Looks Back One Place: If a condition depends only on the previous digit or the current position, keep one count for each possible last digit and update the row; this is a recursion even when no formula is written.

UNIT 13 Pigeonhole, Invariants and Colouring

HIGH WEIGHTAGE

- **Pigeonhole Principle:** $n + 1$ objects in n boxes; **General form:** $kn + 1$ objects force a box with at least $k + 1$; **Choosing the boxes:** residues, pairs with a fixed sum, chains $x, 2x, 4x, \dots$
- **Applications:** Extremal questions with both a bound and an example; Deleting the fewest numbers so that no two remaining numbers add to a prime, or no number is a product of two others (archive).
- **Parity and Colouring:** Chessboard colouring; **Even counts in every row and column** (IOQM 2026 Q12); Colouring segments of a figure under conditions (IOQM 2026 Q23).
- **Invariants:** A quantity unchanged by every allowed operation (parity, a sum modulo k); Proving that a position cannot be reached.
- **Monovariants:** A quantity that only increases or only decreases; **Least number of turns**, found from how much one turn can change it (archive: chips on an 8×5 board).
- **Extremal Principle:** Consider the largest, smallest, first or last object; Medians and order arguments in tables (archive: 3×3 table of 1 to 9).
- **Combinatorial Geometry:** Points, segments and polygons; **Paths through the vertices of a regular polygon that do not cross themselves** (IOQM 2026 Q18); Diagonals and lattice configurations.

HIGH-PRIORITY · PIGEONHOLE RESULTS

Formula Bank:

- Among any $n + 1$ integers, two leave the same remainder on division by n .
- Among any $n + 1$ numbers chosen from $\{1, 2, \dots, 2n\}$, two are consecutive, and one divides another.
- To find both the heaviest and the lightest of n coins on a balance, $\lceil \frac{3n}{2} \rceil - 2$ weighings are needed and enough (88 for 60 coins, Leningrad 1985).

PRO-TIP · EVERY BOUND HAS TWO HALVES

Bound and Example: A “least” or “greatest” question has two parts: a reason why nothing better is possible and an example that reaches the bound. IOQM only marks the number, but the example is how you know the number is right.

Colour First: Colour the board before counting. A chessboard colouring or a colouring in three colours often shows at once why a tiling or a route is impossible.

UNIT 14 Graphs, Games and Probability

- **Graphs:** Vertices, edges and degrees; **Handshake lemma:** the degrees add to twice the number of edges; Complete graph K_n with $\binom{n}{2}$ edges; Paths, cycles and connectedness; Trees on n vertices have $n - 1$ edges.
- **Colouring the Edges:** Two-colourings of the edges of K_n ; A monochromatic triangle in every two-colouring of K_6 ; **Edge conditions that split the points into groups** (IOQM 2026 Q26).
- **Bipartite Graphs and Matchings:** Graphs whose vertices split into two sides; Giving each object a different partner (archive: AIMO 2022, cards and cousins).
- **Board Graphs:** Knight moves and covering a board (archive: ARML 2013, knights on a 4×6 board).
- **Games:** Winning and losing positions; Strategy by symmetry or by pairing; **What a player can guarantee** against any play (archive: painting game on a 7×7 board).
- **Probability:** Classical probability as favourable over total outcomes; Complementary events; Independent events; Conditional probability.
- **Expected Value:** Expectation as a weighted average; Linearity of expectation.

HIGH-PRIORITY · GRAPH FACTS

Formula Bank:

- $\sum \deg v = 2E$, so the number of vertices of odd degree is even.
- K_n has $\frac{n(n-1)}{2}$ edges; a tree on n vertices has $n - 1$ edges.
- Every two-colouring of the edges of K_6 has a triangle of one colour; K_5 can avoid it.
- A connected graph has a closed route using every edge once if and only if every degree is even.

PRO-TIP · TURN THE WORDS INTO A GRAPH

People, Matches, Coloured Segments: When a question speaks of people who know each other, matches played or segments coloured red and blue, draw it as a graph. The conditions usually become statements about degrees; in IOQM 2026 Q26 every point has exactly three red segments.

Probability Is Counting: In IOQM a probability question is a counting question in disguise. Count the total and the favourable outcomes with Units 11 and 12.

[AREA 4 OF 4]

Number Theory *units 15 to 19*

UNIT 15 Divisibility, Primes and Divisors

HIGH WEIGHTAGE

- **Divisibility:** Definition and properties; If $a \mid b$ and $a \mid c$ then $a \mid bx + cy$; **The division algorithm** $a = bq + r, 0 \leq r < b$ (IOQM 2026 Q2).
- **Primes:** Infinitely many primes; Sieve of Eratosthenes; Testing up to $\sqrt{n}$; **2 is the only even prime**; Every prime $p > 3$ is $6k \pm 1$, so $24 \mid p^2 - 1$ (IOQM 2026 Q25).
- **Fundamental Theorem of Arithmetic:** Unique factorisation into primes; Comparing exponents to test squares and cubes, to compare numbers and to count.
- **Divisor Functions:** Number of divisors $\tau(n)$; Sum of divisors $\sigma(n)$; Product of divisors $n^{\tau(n)/2}$; Divisors in pairs $d \leftrightarrow \frac{n}{d}$ (archive: divisors of 11!).
- **Factorials:** **Legendre's formula** for the power of a prime in $n!$; Trailing zeros; Equations such as $10! = a!b!c!$ (archive).
- **Primes in Expressions:** **Factorise to show an expression is composite:** $n^4 + 4, n^4 - 3n^2 + 9 = (n^2 - 3n + 3)(n^2 + 3n + 3)$; Equations in primes such as $p(q - r) = q + r$; Primes of a special form.

HIGH-PRIORITY · DIVISOR FORMULAS

Formula Bank: For $n = p_1^{a_1} p_2^{a_2} \cdots p_k^{a_k}$,

- $\tau(n) = (a_1 + 1)(a_2 + 1) \cdots (a_k + 1)$
- $\sigma(n) = \frac{p_1^{a_1+1} - 1}{p_1 - 1} \cdots \frac{p_k^{a_k+1} - 1}{p_k - 1}$
- n is a perfect square $\iff$ every a_i is even $\iff \tau(n)$ is odd.
- Legendre: the power of p in $n!$ is $\left\lfloor \frac{n}{p} \right\rfloor + \left\lfloor \frac{n}{p^2} \right\rfloor + \left\lfloor \frac{n}{p^3} \right\rfloor + \cdots$
- For a prime $p > 3$: $p^2 \equiv 1 \pmod{24}$.

PRO-TIP · EQUATIONS IN PRIMES

Small Primes First: Try $p = 2$ and $p = 3$ before anything else. After that every prime is odd and of the form $6k \pm 1$, and parity or a remainder modulo 3 removes most of the remaining cases. Many of the prime questions in the archive open exactly this way.

Factorise Before Testing: An expression that splits into two factors greater than 1 is not prime, so it is prime only when the smaller factor equals 1.

UNIT 16 GCD, LCM and Linear Diophantine Equations

- **GCD and LCM:** Definitions; From prime exponents (minimum and maximum); $\gcd(a, b) \cdot \text{lcm}(a, b) = ab$; **GCD and LCM conditions on three numbers** (archive: Cuba 2005).
- **The Euclidean Algorithm:** $\gcd(a, b) = \gcd(b, a - qb)$; **GCD of neighbouring terms of a sequence**, such as $\gcd(n^2 + 2n + 50, (n + 1)^2 + 2(n + 1) + 50)$ (archive).
- **Bezout and Euclid's Lemma:** $ax + by = \gcd(a, b)$ has integer solutions; If $\gcd(a, b) = 1$ and $a \mid bc$, then $a \mid c$.
- **The Equation** $ax + by = c$: Solvable if and only if $\gcd(a, b) \mid c$; General solution $x = x_0 + \frac{b}{d}t, y = y_0 - \frac{a}{d}t$; **Counting or optimising positive solutions** (archive: ARML 2020, $11x + 21y = 2020$).
- **The Coin Problem:** For coprime a, b , the largest number not of the form $ax + by$ with $x, y \geq 0$ is $ab - a - b$.
- **LCM of $1, 2, \dots, n$:** It changes only when n is a prime power, and equals the product of the largest prime powers up to n (archive: AMO 2001, Hong Kong 2018).

HIGH-PRIORITY · GCD AND LCM RESULTS

Formula Bank:

- $\gcd(a, b) \cdot \text{lcm}(a, b) = ab$; consecutive integers are coprime.
- $\gcd(a^m - 1, a^n - 1) = a^{\gcd(m, n)} - 1$ for an integer $a \geq 2$.
- Coin problem: for coprime a, b , exactly $\frac{(a-1)(b-1)}{2}$ positive integers are not of the form $ax + by$ with $x, y \geq 0$, the largest being $ab - a - b$.

PRO-TIP · ONE PRIME AT A TIME

Write the Exponents: When gcd and lcm conditions are mixed, write every number as $2^a 3^b 5^c \dots$ and treat each prime on its own. Each prime gives a small independent puzzle in the exponents.

Neighbouring Terms: For $\gcd(f(n), f(n+1))$, subtract to lower the degree, then multiply to clear the leading coefficient. The gcd ends up dividing a fixed number.

UNIT 17 Congruences and Modular Arithmetic

HIGH WEIGHTAGE

- **Congruences:** $a \equiv b \pmod{n}$; Adding, subtracting, multiplying and raising to powers; **Cancelling only a factor coprime to n .**
- **Linear Congruences:** $ax \equiv b \pmod{n}$ is solvable if and only if $\gcd(a, n) \mid b$; Inverses modulo n .
- **Chinese Remainder Theorem:** Systems with coprime moduli; **Splitting a modulus**, such as $1000 = 8 \times 125$ (archive: $n^2 \equiv n \pmod{1000}$).
- **Fermat and Wilson:** **Fermat's little theorem:** $a^{p-1} \equiv 1 \pmod{p}$ when $p \nmid a$; **Wilson's theorem:** $(p-1)! \equiv -1 \pmod{p}$.
- **Euler's Theorem:** $\varphi(n) = n \prod \left(1 - \frac{1}{p}\right)$; $a^{\varphi(n)} \equiv 1 \pmod{n}$ when $\gcd(a, n) = 1$.

- **Order and Cycles:** The least k with $a^k \equiv 1$; **The order divides every exponent that gives 1**; Cycles of last digits; For $\gcd(n, 10) = 1$, the decimal of $\frac{1}{n}$ repeats with period equal to the order of 10 modulo n (archive: AIMO 2023).
- **Huge Exponents:** Reducing a^N with a known $a^k \equiv 1$; **Moduli of the form $\frac{2^k-1}{3}$ and 2^k-1** (IOQM 2026 Q29).
- **Powers Modulo Small Numbers:** Squares modulo 3, 4, 5, 8; Cubes modulo 7 and 9; **Showing an equation has no solution by one modulus** (archive: $x^2 + y^2 = 3z^2$).
- **Last Digits:** Last one or two digits of a^n from the cycle of powers or from the binomial theorem.

HIGH-PRIORITY · RESIDUES WORTH MEMORISING

Formula Bank:

- Squares: modulo 3 are 0, 1; modulo 4 are 0, 1; modulo 5 are 0, 1, 4; modulo 8 are 0, 1, 4, and an odd square is 1 (mod 8).
- Cubes: modulo 7 are 0, 1, 6; modulo 9 are 0, 1, 8. Fourth powers modulo 16 are 0, 1.
- Fermat: $a^p \equiv a \pmod{p}$. Euler: $a^{\varphi(n)} \equiv 1 \pmod{n}$. Wilson: $(p-1)! \equiv -1 \pmod{p}$.

PRO-TIP · CHOOSING THE MODULUS

Few Residues: To rule out an equation, choose a modulus where its powers have few values: 3 or 4 for squares, 7 or 9 for cubes, 16 for fourth powers. The archive's $x^2 + y^2 = 3z^2$ has no positive solution: modulo 3 forces $3 \mid x$ and $3 \mid y$, and descent finishes it.

Cycle Length First: For a huge power, find the length k of the cycle of the base, then reduce the exponent modulo k . A remainder of 0 means the last term of the cycle, not the first.

UNIT 18 Digits, Number Bases and the Floor Function

HIGH WEIGHTAGE

- **Place Value:** $\overline{abc} = 100a + 10b + c$; **Equations in the digits** (IOQM 2026 Q6); Reversal: $\overline{abc} - \overline{cba} = 99(a - c)$ (IOQM 2026 Q14).
- **Digit Sums:** $S(n) \equiv n \pmod{9}$; **The smallest number with a given digit sum** (IOQM 2026 Q9); Digit sum of $9n$ when the digits of n increase (archive: Russia 1999).
- **Divisibility Tests:** Tests for 2, 3, 4, 5, 8, 9, 11; 7, 11 and 13 through $1001 = 7 \cdot 11 \cdot 13$; 37 through $111 = 3 \cdot 37$.
- **Moving Digits:** **A digit moved from the front to the back** (IOQM 2026 Q20); Numbers formed by inserting or deleting a digit.
- **Number Bases:** Writing a number in base b and converting between bases; Binary form and sums of distinct powers of 2 (archive: $2^x + 2^y + 2^z = 2336$); Numbers with the same number of digits in two bases.
- **Decimals:** Terminating and recurring decimals; Period of the decimal of $\frac{1}{n}$ (Unit 17).
- **Floor and Fractional Part:** $\lfloor x \rfloor \leq x < \lfloor x \rfloor + 1$ and $\{x\} = x - \lfloor x \rfloor$; Multiples of k up to n : $\lfloor \frac{n}{k} \rfloor$; **Equations with floors**, such as $n + \lfloor \sqrt{n} \rfloor + \lfloor \sqrt[4]{n} \rfloor = 2017$ (archive).

HIGH-PRIORITY · DIGIT FACTS

Formula Bank:

- $n \equiv S(n) \pmod{9}$; $n \equiv (\text{alternating sum of digits from the units digit}) \pmod{11}$.
- $\overline{abc} - \overline{cba} = 99(a - c)$ and $\overline{ab} + \overline{ba} = 11(a + b)$.
- Smallest number with digit sum $s = 9q + r$, $0 < r < 9$: the digit r followed by q nines.
- A positive integer N has $\lfloor \log_b N \rfloor + 1$ digits in base b .

PRO-TIP · TURN THE DIGITS INTO AN EQUATION

Place Values at Once: Write the number with its place values and the condition becomes an equation in the digits. The bounds $0 \leq \text{digit} \leq 9$ and a non-zero leading digit then leave very few cases.

Cyclic Shift: If a leading 3 is moved to the end and x is the number formed by the other k digits, then $M = 3 \cdot 10^k + x$ and the new number is $10x + 3$.

UNIT 19 Diophantine Equations and Pythagorean Triples

HIGH WEIGHTAGE

- **Factoring Methods: Simon's factoring:** $xy + ax + by = c \iff (x + b)(y + a) = c + ab$; Difference of squares $a^2 - b^2 = (a - b)(a + b)$, with both factors of the same parity; Factorising a quadratic form, $2x^2 - 11xy + 5y^2 = (2x - y)(x - 5y)$ (archive: Baltic Way 1998).
- **Perfect Squares: Complete the square, then factorise:** $x^2 + 84x + 2008 = y^2$ becomes $(y - x - 42)(y + x + 42) = 244$ (archive: AIME 2008, RMO 1992); **Between consecutive squares:** if $k^2 < N < (k + 1)^2$, then N is not a square (archive: APMO 2011); Squares modulo small numbers (Unit 17).
- **Quadratic in One Unknown:** Treat the equation as a quadratic in one variable; **the discriminant must be a perfect square** (archive: Ecuador 2017, $p^2 - p + 1$ a perfect cube).
- **Sums of Two Squares:** Which numbers are $a^2 + b^2$ (IOQM 2026 Q10); A prime $p \equiv 3 \pmod{4}$ dividing $a^2 + b^2$ divides both a and b ; $(a^2 + b^2)(c^2 + d^2) = (ac - bd)^2 + (ad + bc)^2$.
- **Pythagorean Triples: Primitive triples** $m^2 - n^2, 2mn, m^2 + n^2$ with $m > n > 0$, $\gcd(m, n) = 1$, $m - n$ odd; In every triple one leg is divisible by 3, one leg by 4 and one side by 5.
- **Equations with Powers and Factorials:** $n! + 8 = 2^k, 3 \cdot 2^m + 1 = n^2, 4^n + 2007$ a square: factorise, then use parity or a modulus (archive).
- **Bounding and Descent:** Size arguments that leave finitely many cases; Infinite descent; **Least value of a sum under a condition** (IOQM 2026 Q15).

PRO-TIP · OPENING A DIOPHANTINE EQUATION

Three Questions in Order: Can it be factorised? Can one variable be bounded? Which modulus forbids most cases? Ask them in this order, and come back to the first after each of the others.

HIGH-PRIORITY · STANDARD FORMS

Formula Bank:

- $xy + ax + by = c \iff (x + b)(y + a) = c + ab$.
- $N = x^2 - y^2$ has an integer solution if and only if $N \not\equiv 2 \pmod{4}$.
- Every primitive triple is $(m^2 - n^2, 2mn, m^2 + n^2)$ with $m > n > 0$, $\gcd(m, n) = 1$ and $m - n$ odd.
- n is a sum of two squares if and only if every prime $p \equiv 3 \pmod{4}$ appears in n to an even power.

FINAL STRATEGY · IOQM 2027 MASTER PLAN

The 2026 Paper by Area: The thirty questions of IOQM 2026, as sorted in our solutions:

- **Number Theory:** 10 questions, 32 marks. **Combinatorics:** 9 questions, 30 marks.
- **Geometry:** 7 questions, 25 marks (three needed trigonometry). **Algebra:** 4 questions, 13 marks.

Tier 1: The “Heavy Hitters” Units 15 to 19 and 11 to 14. Number theory and combinatorics carried 62 of the 100 marks in 2026. Mastery here builds the base score.

Tier 2: Geometry Units 06 to 09. Angle chasing, circles and the sine and cosine rules; every geometry question of 2026 came from these four units.

Tier 3: Algebra Units 01 to 05. Fewer questions carry the label, but the identities of Unit 01 are used inside questions of every area.

Ritesh Sir Advice: Solve each past IOQM paper in one sitting of three hours, then mark every question against the unit it came from. The units where you lost marks decide your next month. Every answer is a whole number from 00 to 99 and is either exactly right or worth nothing, so check each final number before you write it.

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